Soluble Groups and p-Groups

Talk 1: Polycyclic Presentations

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S_4

```
gap> G := SymmetricGroup(4);
Sym( [ 1 .. 4 ] )
gap> GeneratorsOfGroup(G);
[ (1,2,3,4), (1,2) ]
gap> iso := IsomorphismPcpGroup(G);
[ (3,4), (2,4,3), (1,4)(2,3), (1,3)(2,4) ] -> [ g1, g2, g3, g4 ]
gap> K := Image(iso);
Pcp-group with orders [ 2, 3, 2, 2 ]
```

S_4

```
gap> PrintPcpPresentation(K);
g1^2 = id
g2^3 = id
g3^2 = id
g4^2 = id
g2 ^ g1 = g2^2
g3 ^ g1 = g4
g3 ^ g2 = g4
g4 ^ g1 = g3
g4 ^ g2 = g3 * g4
```

S_4

```
gap> D := DerivedSubgroup(K);
Pcp-group with orders [ 3, 2, 2 ]
gap> Igs(D);
[ g2, g3, g4 ]
gap> Center(K);
Pcp-group with orders [  ]
```

D_∞

```
gap> G := DihedralPcpGroup(0);
Pcp-group with orders [ 2, 0 ]
gap> PrintPcpPresentation(G);
g1^2 = id
g2 ^ g1 = g2^-1
```

D_∞

```
gap> D := DerivedSubgroup(G);
Pcp-group with orders [ 0 ]
gap> G/D;
Pcp-group with orders [ 2, 2 ]
gap> IsAbelian(D);
true
gap> AbelianInvariants(D);
[ 0 ]
```

Matrixgroups

```
gap> mats :=
>   [ [ [ 1, 1, 0 ], [ 0, 1, 0 ], [ 0, 0, 1 ] ],
>     [ [ 1, 0, 0 ], [ 0, 1, 1 ], [ 0, 0, 1 ] ],
>     [ [ 1, 0, 1 ], [ 0, 1, 0 ], [ 0, 0, 1 ] ] ];;
gap> G := Group(mats);
<matrix group with 3 generators>
gap> Size(G);
infinity
gap> IsNilpotent(G);
... Error message ...
```

Matrixgroups

```
gap> IsomorphismPcpGroup(G);
[ [ [ 1, 1, 0 ], [ 0, 1, 0 ], [ 0, 0, 1 ] ],
  [ [ 1, 0, 0 ], [ 0, 1, 1 ], [ 0, 0, 1 ] ],
  [ [ 1, 0, 1 ], [ 0, 1, 0 ], [ 0, 0, 1 ] ] ] -> [ g1, g2, g3 ]
gap> K := Image(last);
Pcp-group with orders [ 0, 0, 0 ]
gap> IsNilpotent(K);
true
gap> Center(K);
Pcp-group with orders [ 0 ]
gap> Igs(last);
[ g3 ]
```

Groups by Collector

```
gap> c := FromTheLeftCollector(2);
<<from the left collector with 2 generators>>
gap> SetRelativeOrder(c, 1, 2);
gap> SetRelativeOrder(c, 2, 3);
gap> SetConjugate(c, 2, 1, [2, 2]);
gap> UpdatePolycyclicCollector(c);
gap> IsConfluent(c);
true
gap> G := PcpGroupByCollector(c);
Pcp-group with orders [ 2, 3 ]
gap> Size(G);
6
gap> IsAbelian(G);
false
```