

Soluble Groups and p-Groups

Talk 2: Finite soluble groups

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A maximal subgroup of F_{22}

```
gap> G;
<pc group of size 5038848 with 17 generators>
gap> Factors(Size(G));
[ 2, 2, 2, 2, 2, 2, 2, 2, 3, 3, 3, 3, 3, 3, 3, 3 ]
gap> S2 := SylowSubgroup(G, 2);
<pc group of size 256 with 8 generators>
gap> IsAbelian(S2);
false
gap> S3 := SylowSubgroup(G, 3);
<pc group of size 19683 with 9 generators>
gap> IsAbelian(S3);
false
```

A maximal subgroup of F_{22}

```

gap> AutomorphismGroup(G);
... takes a little bit of time ...
<group of size 90699264 with 7 generators>
gap> time;
67593
gap> GeneratorsOfGroup(A)[1];
Pcgs([ f1, f2*f4*f5*f6*f17^2, f3*f9, f4*f7*f9*f10*f17^2, f5*f6*f
f6*f17^2, f7*f17, f8*f9*f10, f9, f10, f11*f12*f15*f16*f17,
f12*f15^2*f16^2*f17, f13*f14*f16^2, f14^2*f15^2*f17^2, f15*f16
f16^2*f17, f17 ]) -> [ f1*f2^2*f5*f8*f9*f17^2, f2^2*f5*f6*f7*f
f3^2*f4*f5*f10, f5*f6*f7*f8*f9*f17^2, f4*f7*f17^2, f7*f8*f10*f
f6*f9*f17^2, f8*f9*f10, f8, f10, f12^2*f13*f14*f16*f17^2, f14^
f12^2*f15*f17, f11*f14^2*f15, f12^2*f14*f17^2, f12^2*f17^2, f1

```

A larger solvable group

```
gap> ReadPackage("format","grp/DARKplus.gi");
#I A group of order 367225562592682565625000000000000000000 has
#I It is called DARKplus
true
gap> G := DARKplus;
<pc group of size 367225562592682565625000000000000000000 with
gap> pr := Set(Factors(Size(G)));
[ 2, 3, 5, 7, 31 ]
gap> S := List(pr, x -> SylowSubgroup(G,x));
[ <pc group of size 524288 with 19 generators>,
  <pc group of size 19683 with 9 generators>,
  <pc group of size 59604644775390625 with 24 generators>,
  <pc group of size 7 with 1 generators>,
  <pc group of size 852891037441 with 8 generators> ]
gap> time;
116
gap> List(S, IsAbelian);
[ false, false, true, true, true ]
```