Soluble Groups and p-Groups

Talk 3: Infinite polycyclic groups

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The infinite dihedral group

```
gap> G := DihedralPcpGroup(0);
Pcp-group with orders [ 2, 0 ]
gap> g := Random(G);
g2^-1
gap> h := Random(G);
g2
gap> IsConjugate(G, g, h);
g1
gap> h := Random(G);
g1*g2^-5
gap> IsConjugate(G, g, h);
false
```

The infinite dihedral group

```
gap> G := DihedralPcpGroup(0);
Pcp-group with orders [ 2, 0 ]
gap> TorsionSubgroup(G);
fail
gap> cl := FiniteSubgroupClasses(G);
[ Pcp-group with orders [ 2 ]^G,
  Pcp-group with orders [ 2 ]^G,
  Pcp-group with orders [  ]^G ]
gap> List(cl, x -> Igs(Representative(x)));
[ [ g1 ], [ g1*g2 ], [  ] ]
gap> List(cl, x -> Size(x));
[ infinity, infinity, 1 ]
```

Groups from algebraic number fields

```
gap> x := Indeterminate(Rationals);;
gap> f := x^4 + 3*x^3 + 7;;
gap> G := MaximalOrderByUnitsPcpGroup(f);
Pcp-group with orders [ 2, 0, 0, 0, 0, 0, 0 ]
```

Groups from algebraic number fields

```
gap> PrintPcpPresentation(G);
g1^2 = id
g4 ^ g1 = g4^-1
g4 ^ g2 = g4^2 * g5
g4 ^ g2^-1 = g4^4 * g5^-2 * g6 * g7
g4 ^ g3 = g4^3 * g5^-3 * g6 * g7
g4 ^ g3^-1 = g4^5 * g5^-2 * g6 * g7^2
g5 ^ g1 = g5^-1
g5 ^ g2 = g5^2 * g6
g5 ^ g2^-1 = g4^-7 * g5^4 * g6^-2 * g7^-2
g5 ^ g3 = g4^-7 * g5^3 * g6^-3 * g7^-2
g5 ^ g3^-1 = g4^-14 * g5^5 * g6^-2 * g7^-5
g6 ^ g1 = g6^-1
g6 ^ g2 = g6^2 * g7
g6 ^ g2^-1 = g4^14 * g5^-7 * g6^4 * g7^4
g6 ^ g3 = g4^14 * g5^-7 * g6^3 * g7^3
g6 ^ g3^-1 = g4^35 * g5^-14 * g6^5 * g7^13
g7 ^ g1 = g7^-1
g7 ^ g2 = g4^-7 * g7^-1
g7 ^ g2^-1 = g4^-28 * g5^14 * g6^-7 * g7^-8
g7 ^ g3 = g4^-21 * g5^14 * g6^-7 * g7^-6
g7 ^ g3^-1 = g4^-91 * g5^35 * g6^-14 * g7^-34
```

Groups from algebraic number fields

```
gap> TorsionSubgroup(G);
fail
gap> FiniteSubgroupClasses(G);
[ Pcp-group with orders [ 2 ]^G,
  Pcp-group with orders [ 2 ]^G,
  Pcp-group with orders [  ]^G ]
gap> g := Random(G);
g1*g2^-2*g3^3*g4^-4*g6^-2*g7^-3
gap> h := Random(G);
g1*g2^-2*g4^2*g5^-1*g6^5*g7^-1
gap> IsConjugate(G, g, h);
false
gap> h := g^Random(G);
g1*g2^-2*g3^3*g4^-18250*g5^10156*g6^-5654*g7^-4683
```

Groups from algebraic number fields

```
gap> IsConjugate(G, g, h);
g2*g3^6824*g4^
-3634075384357370595587318236427324011387885754624728022165198081386998093977124512862399694701073176049458104681163007032843166535169543414300356043994365462011486021578040727745335690261121099796925390197515654137606006654452185070305701059235915202955051908147350087447159529040511383767967207224652 ....
... several pages of output ...
gap> time;
111015
```

Space Groups

```
gap> G := SpaceGroupPcpGroup( 4, 200 );
Pcp-group with orders [ 2, 2, 2, 0, 0, 0, 0 ]
gap> T := FittingSubgroup(G);
Pcp-group with orders [ 0, 0, 0, 0 ]
gap> ComplementClasses(G, T);
[  ]
gap> FiniteSubgroupClasses(G);
[ Pcp-group with orders [ 2, 2 ]^G, Pcp-group with orders [ 2, 2
  Pcp-group with orders [ 2, 2 ]^G, Pcp-group with orders [ 2, 2
  Pcp-group with orders [ 2 ]^G, Pcp-group with orders [ 2 ]^G,
  Pcp-group with orders [ 2 ]^G, Pcp-group with orders [ 2 ]^G,
  Pcp-group with orders [ 2 ]^G, Pcp-group with orders [ 2 ]^G,
  Pcp-group with orders [ 2 ]^G, Pcp-group with orders [ 2 ]^G,
  Pcp-group with orders [ 2 ]^G, Pcp-group with orders [ 2 ]^G,
  Pcp-group with orders [ 2 ]^G, Pcp-group with orders [  ]^G ]
```